magnetic Energy: $U = -\vec{m}\cdot\vec{B}$

Force experienced by magnetic dipole:

$$\vec{F} = \nabla(\vec{m}\cdot\vec{B})$$

For z orientation

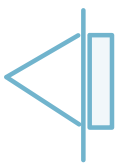
$$F_z = m_z \cdot \frac{\partial}{\partial z} B_z$$

$$\begin{cases} \text{if } m_z > 0 \longrightarrow Ag \nearrow \\ \text{if } m_z < 0 \longrightarrow Ag \swarrow \end{cases} \Rightarrow$$

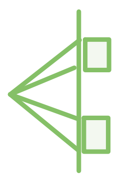
SG measures the z-component of $\vec{m}$

$m_z \in \{-|\vec{m}|, |\vec{m}|\}$

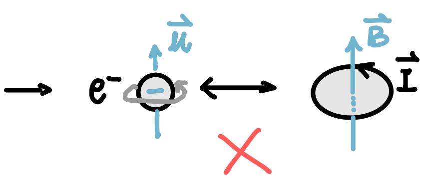
$\Rightarrow$ classic perspective

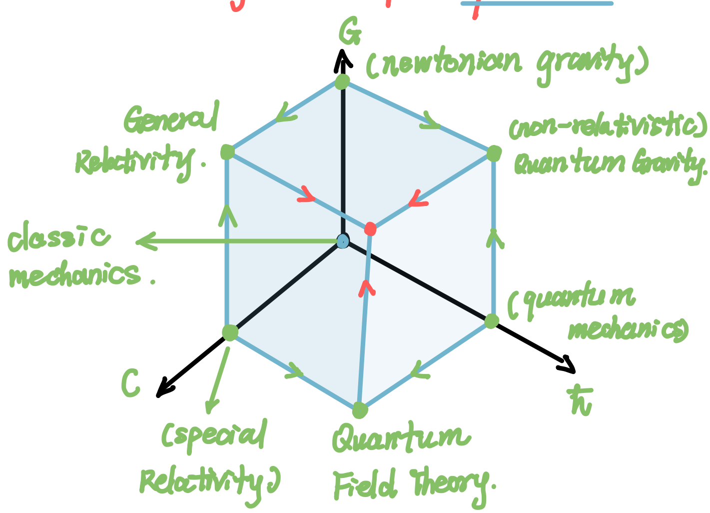


However



*Q1: what's the origin of this magnetic dipole quanta?



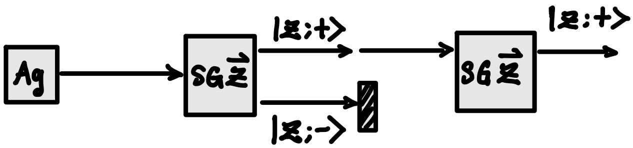


$\left.\begin{matrix}\sigma^+ \\ \sigma^-\end{matrix}\right\} \dashrightarrow$ Basis for light.

Angular moment.

* Q2: What's observation process.

1>

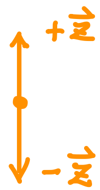


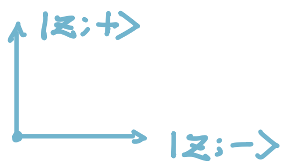
$\Rightarrow |\langle z;-|z;+\rangle|^2 = 0$ $\quad\quad$ $\Rightarrow |\langle z;+|z;+\rangle|^2 = 1$

$\Rightarrow \langle z;-|z;+\rangle = 0$ $\quad\quad$ $|\langle z;-|z;-\rangle|^2 = 1$

orthogonal.

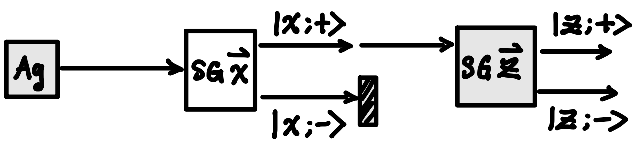
Real space $\quad\quad$ Quantum Space.





*. same conditions for $x-$ & $y-$ apparatus / observation.

2>



$P_+ = 50\%$

$P_- = 50\%$

$\Rightarrow |\langle z;+|x;+\rangle|^2 = 1/2$

$|\langle z;-|x;+\rangle|^2 = 1/2$

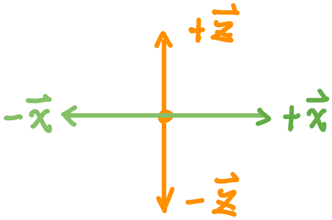
$\Rightarrow |x;+\rangle = C_1 |z;+\rangle + C_2 |z;-\rangle$

$|x;-\rangle = C_3 |z;+\rangle + C_4 |z;-\rangle$

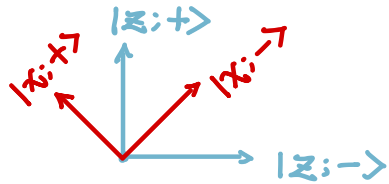
$|x;+\rangle = \frac{1}{\sqrt{2}}|z;+\rangle + \frac{1}{\sqrt{2}}|z;-\rangle$

$|x;-\rangle = \frac{1}{\sqrt{2}}|z;+\rangle - \frac{1}{\sqrt{2}}|z;-\rangle$

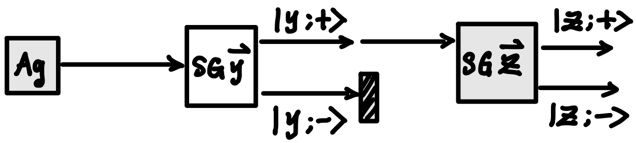
Real space



Quantum Space.



3>



$P_+ = 50\%$

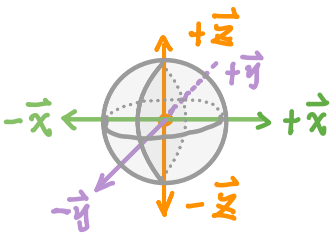
$P_- = 50\%$

$$\Rightarrow \left.\begin{aligned} |y;+\rangle &= C_5|z;+\rangle + C_6|z;-\rangle \\ |y;-\rangle &= C_7|z;+\rangle + C_8|z;-\rangle \end{aligned}\right\} \Rightarrow \begin{aligned} \{C_5, C_6\} &\neq \{C_1, C_2\} \\ \{C_7, C_8\} &\neq \{C_3, C_4\} \end{aligned}$$

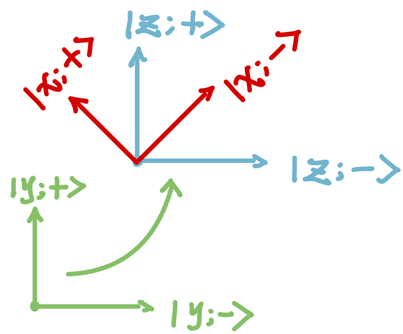
$$\Rightarrow |y;+\rangle = \frac{1}{\sqrt{2}}|z;+\rangle + \frac{i}{\sqrt{2}}|z;-\rangle$$

$$|y;-\rangle = \frac{1}{\sqrt{2}}|z;+\rangle - \frac{i}{\sqrt{2}}|z;-\rangle$$

Real space (Bloch sphere)



Quantum Space.



Now assign eigen basis vectors.

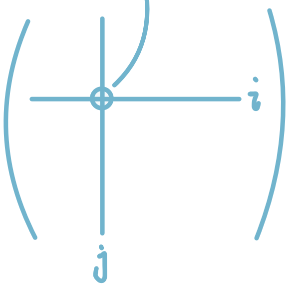
$$|z;+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |z;-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \quad |x;+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad |x;-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad |y;+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |y;-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Q3. What's the nature of the observation itself.? (operator)?

1> $\langle a_i | \hat{A} | a_j \rangle$



2> $|a_i\rangle\langle a_j| = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & 1 & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$

3> $$\sum_i \sum_j |a_i\rangle\langle a_j| \underbrace{(\langle a_i | \hat{A} | a_j \rangle)}_{\text{scalar.}}$$

$$= \hat{A}$$

$$= \sum_i \sum_j |a_i\rangle \underbrace{\langle a_i | \underbrace{\hat{A} | a_j\rangle}_{a_j |a_j\rangle}}_{\underbrace{a_j \langle a_i | a_j \rangle}_{\delta_{ij}}} \langle a_j |$$

$$= \underbrace{\sum_i a_i |a_i\rangle\langle a_i|}_{\hat{A}}$$