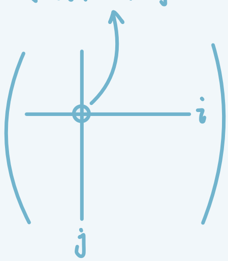
1-3. Energy eigen value & eigen ket.

suppose base kets $\{|a'\rangle\}$ $\quad \hat{A}|a'\rangle = a'|a'\rangle$

& $[\hat{A}, \hat{H}] = 0 \Rightarrow \hat{H}|a'\rangle = \underbrace{E_{a'}}_{\text{Energy eigenvalue.}}\underbrace{|a'\rangle}_{\text{Energy eigen ket.}}$

1> $\langle a_i|\hat{A}|a_j\rangle$



2> $|a_i\rangle\langle a_j| = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & 1 & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$

3> $\sum_i \sum_j |a_i\rangle\langle a_j| \underbrace{(\langle a_i|\hat{A}|a_j\rangle)}_{\text{scalar.}}$

$= \hat{A}$

$= \sum_i \sum_j |a_i\rangle\langle a_i|\underbrace{\hat{A}|a_j\rangle}_{a_j|a_j\rangle}\langle a_j|$

$\underbrace{a_j\langle a_i|a_j\rangle}_{\delta_{ij}}$

$= \underbrace{\sum_i a_i |a_i\rangle\langle a_i|}_{\hat{A}}$

$\Rightarrow \hat{u} = e^{-\frac{i}{\hbar}\hat{H}(t-\underset{0}{t_0})} = e^{-\frac{i}{\hbar}\hat{H}t}$ ⟶ Energy eigenvalues?

$= \sum_{a'}\sum_{a''} |a''\rangle\langle a''|\, e^{-\frac{i}{\hbar}\hat{H}t}\, |a'\rangle\langle a'|$

$e^{-\frac{i}{\hbar}\hat{H}t} = \sum_{n=0}^{\infty} \frac{(-\frac{i}{\hbar}\hat{H}t)^n}{n!} \Rightarrow \underbrace{\sum_{n=0}^{\infty} \frac{(-\frac{i}{\hbar}E_{a'}t)^n}{n!}}_{e^{-\frac{i}{\hbar}E_{a'}t}} |a'\rangle$

$$\Downarrow$$

$$e^{-\frac{i}{\hbar}E_{a'}t}\,\delta_{a'a''}$$

$$= \sum_{a'}\sum_{a''} |a''\rangle\, e^{-\frac{i}{\hbar}E_{a'}t}\,\delta_{a'a''}\,\langle a'|$$

$$= \boxed{\sum_{a'} |a'\rangle\, e^{-\frac{i}{\hbar}E_{a'}t}\,\langle a'|} \quad *.$$

significance: if $E_{a'}$ is known, and expansion of $|\alpha, t_0\rangle$ in $\{|a'\rangle\}$ is known (i.e., $|\alpha, t_0\rangle = \sum_{a'} |a'\rangle \underbrace{\langle a'|\alpha, t_0\rangle}_{C_{a'}}$)

$\Rightarrow$ $|\alpha, t_0; t\rangle$ is then known.

$$|\alpha, \underset{\substack{\parallel \\ 0}}{t_0}; t\rangle = \hat{U}(t, 0)\,|\alpha, t_0\rangle = e^{-\frac{i}{\hbar}\hat{H}t}\,|\alpha, t_0\rangle$$

$$\Downarrow$$

$$\underbrace{\sum_{a'} |a'\rangle\, e^{-\frac{i}{\hbar}E_{a'}t}\, \underbrace{\langle a'|\alpha, t_0\rangle}_{C_{a'}}}_{C_{a'}(t)}$$

$$\Rightarrow \sum_{a'} C_{a'}(t)\,|a'\rangle$$

if $|\alpha, t_0\rangle = |a'\rangle$ $\Rightarrow$ $|\alpha, t_0; t\rangle = |a', t_0; t\rangle$

$$= e^{-\frac{i}{\hbar}E_{a'}t}\,|a'\rangle$$

$\Rightarrow$ initially $\hat{H}|a'\rangle = E_{a'}|a'\rangle$

$\hat{A}\,|a'\rangle = a'\,|a'\rangle$

time evolves: $\underline{\hat{H}\,|a', t_0; t\rangle} = \hat{H}\, e^{-\frac{i}{\hbar}E_{a'}t}\,|a'\rangle$

(1) $|a', t_0; t\rangle$ is still the eigenket of $\hat{H}$

(2) Energy eigenvalue is t-independent.

$\Leftarrow$

$= E_{a'} \boxed{e^{-\frac{i}{\hbar}E_{a'}t}} |a'\rangle$

$= E_{a'} |a', t_0; t\rangle$

* Phase modulation.

1-4. time-dependent Expectation value.

$\hat{A} \quad |\alpha\rangle \quad \Rightarrow \quad \langle \hat{A} \rangle_\alpha \equiv \langle \alpha | \hat{A} | \alpha \rangle$

1) if $\hat{A}|a'\rangle = a'|a'\rangle$, $\{|a'\rangle\}$ is the basis set.

$[\hat{A}, \hat{H}] = 0$, $\Rightarrow$ sys is in $\underline{|a', t_0; t\rangle} = e^{-\frac{i}{\hbar}E_{a'}t} |a'\rangle$

$\downarrow$

$\hat{H}|a'\rangle = E_{a'}|a'\rangle$

$$\Rightarrow \langle \hat{B} \rangle_{a', t} = \langle a', t_0; t | \hat{B} | a', t_0; t \rangle$$

$$= \langle a', t_0 | \hat{u}^\dagger(t, 0)\, \hat{B}\, \hat{u}(t, 0) | a', t_0 \rangle$$

$$= \langle a', t_0 | \cancel{e^{\frac{i}{\hbar}E_{a'}t}}\, \hat{B}\, \cancel{e^{-\frac{i}{\hbar}E_{a'}t}} | a', t_0 \rangle$$

$$= \langle a', t_0 | \hat{B} | a', t_0 \rangle \rightarrow \text{t- independent.}$$

$\hat{B}$ itself is t-independent.

$\Rightarrow$ even though $|a', t_0; t\rangle$, the expectation value is t-independent.

stationary.

Energy eigen state ($|a'\rangle$) $\rightarrow$ stationary state.

2> if sys is in $|\alpha, t_0\rangle = \sum_{a'} C_{a'} |a'\rangle$

$$\Rightarrow \langle \hat{B} \rangle_{\alpha, t} = \langle \alpha, t_0 ; t | \hat{B} | \alpha, t_0 ; t \rangle$$

$$= \left[\sum_{a'} C^*_{a'} e^{\frac{i}{\hbar} E_{a'} t} \langle a' | \right] \hat{B} \left[\sum_{a''} C_{a''} e^{-\frac{i}{\hbar} E_{a''} t} |a''\rangle \right]$$

$$= \left[\sum_{a'} C^*_{a'} e^{\frac{i}{\hbar} E_{a'} t} \langle a' | \right] \left[\sum_{a''} C_{a''} e^{-\frac{i}{\hbar} E_{a''} t} \underbrace{\hat{B} |a''\rangle}_{\text{ket vector}} \right]$$

$$= \sum_{a'} \sum_{a''} C^*_{a'} C_{a''} \underbrace{\langle a' | \hat{B} | a'' \rangle}_{\text{matrix element}} \underbrace{e^{-\frac{i}{\hbar} (E_{a''} - E_{a'}) t}}_{\text{oscillation term.}}$$

$e^{-i\omega t}$, where $\omega = \frac{E_{a''} - E_{a'}}{\hbar}$

Bohr's Frequency

1> Diagonal term:

$a' = a''$

$$\Rightarrow e^{-\frac{i}{\hbar} (E_{a''} - E_{a'}) t} = 1$$

$$\Rightarrow \langle \hat{B} \rangle_{\alpha, t} = \underbrace{\sum_{a'} |C_{a'}|^2 \langle a' | \hat{B} | a' \rangle}_{\text{t-independent.}}$$

(static contribution to expectation value).

2> off-diagonal term: $a' \neq a''$

oscillatory term: $e^{-\frac{i}{\hbar} (E_{a''} - E_{a'}) t} \langle a' | \hat{B} | a'' \rangle$

Frequency: $W_{a'-a''} = \frac{E_{a'} - E_{a''}}{\hbar}$.

*. if $|a'\rangle = |g\rangle$, $|a''\rangle = |e\rangle$

⇒ an coherent electronically excited state is a dynamic state that oscillates between the ground & excited state.

pump – probe

1> Pump → excite the sys. into a coherent superposition.

2> probe measure how sys elvoves

3> the detected signal oscillates in time.

1-5. spin precession. $(|\alpha, t_0; t\rangle)$

$\hat{H}$ for spin ½ sys.

$\downarrow$

(Energy)

magnetic energy $E = -\vec{\mu} \cdot \vec{B}$

$\vec{\mu}$ ↓ magnetic dipole moment.

$\vec{\mu} = \gamma \cdot \vec{S}$

$\vec{S}$ ↓ $\pm\frac{\hbar}{2}$: Angular momentum of particle.

γ ↓ gyromagnetic ratio

classic: related to particle's charge & mass.

by $\gamma = \frac{q}{2mc}$

more quantum: for e− $\gamma = -\frac{e}{2m_e c} \cdot g_e$

g_e ↑ g-factor (~2) (Landé)

$\Rightarrow E = -\vec{\mu} \cdot \vec{B} = -\gamma \cdot \vec{S} \cdot \vec{B}$

$\Rightarrow \hat{H} = -\gamma \cdot \hat{S} \cdot \vec{B} \longrightarrow (B_x, B_y, B_z)$

$\hat{S}$ ↓ $(\hat{S}_x, \hat{S}_y, \hat{S}_z)$