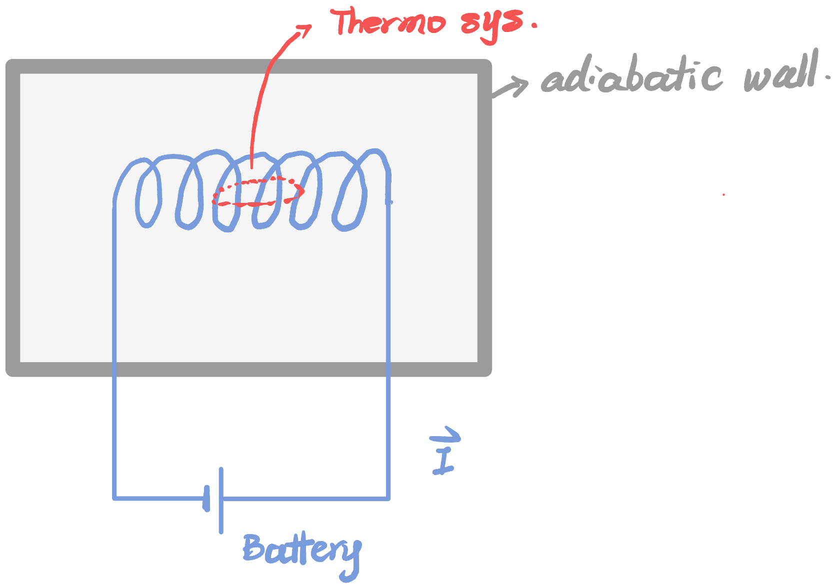

$U = U(S, V, N, \underline{m})$ (Paramagnetic Sys)

$\rightarrow \underline{\left(\frac{\partial U}{\partial m}\right)_{S,V,N}} = P_j$ $\vec{m}$: [J/T] (Ising model)

$\downarrow$ $\downarrow$

$B_e \rightarrow$ Tesla [T] Ferromag

Anti-Ferro

$dU = TdS - PdV + \mu dN + \vec{B_e} \cdot d\vec{I}$

$\underline{U} \equiv$ Energy $-$ $\underline{\text{(vacuum magnetostatic Energy)}}$

$\downarrow$

$\underline{\text{Battery = current} \times \text{voltage.}}$

magnetostatic $\underline{\text{energy density}}$: $\frac{1}{2} \frac{1}{\mu_0} B_e^2$

$\downarrow$ $\downarrow$

Energy $= \int \frac{1}{2} \frac{1}{\mu_0} B_e^2 \, dV$ vacuum magnetic permeability.

$[T \cdot m \cdot A^{-1}] \rightarrow 4\pi \times 10^{-7}$

4-6. Heat capacity and other derivations.

For ideal gas : P, V, T.

cyclic rule: $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$

$\rightarrow \left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$

$$= -\frac{\boxed{\left(\frac{\partial V}{\partial T}\right)_P}}{\boxed{\left(\frac{\partial V}{\partial P}\right)_T}}$$

$\left(\frac{\partial V}{\partial T}\right)_P$ → How volume changes by T

$\left(\frac{\partial V}{\partial P}\right)_T$ → How volume changes by P.

→ $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$: thermal expansion coefficient.

→ $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$: isothermal compressibility.

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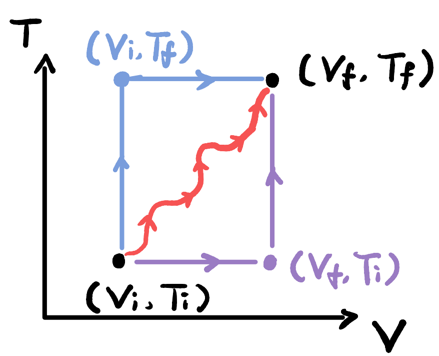
*. Dependence of U on (P, V, T)
(two of them)

$U = U(V, T)$

$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$



since: $dU = đQ + đW$

$= đQ - PdV$

$$\rightarrow \left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{đQ}{\partial T}\right)_V = C_V$$

$$\rightarrow \quad dU = C_V \cdot dT + \underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{?\ T\left(\frac{\partial P}{\partial T}\right)_V - P} dV$$

Derivation

1> $$dS = \underbrace{\frac{1}{T}}_{F_{j=0}} dU + \underbrace{\frac{P}{T}}_{F_{j=0} = -\frac{P_j}{T}} dV \qquad -P$$

$$= \frac{1}{T}\left[C_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV\right] + \frac{P}{T} dV$$

$$= \frac{C_V}{T} dT + \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

$S = S(V, T)$

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$\Rightarrow \quad \left(\frac{\partial S}{\partial T}\right)_V = \frac{C_V}{T} \quad \& \quad \left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T}\left[\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{?} + P\right]$$

2> Because of the state-function nature of S.

$$\left(\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V$$

$$\left(\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial V}\left(\frac{C_V}{T}\right)\right)_T = \underline{\frac{1}{T}\left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right)_T}$$

$$\parallel$$

$$\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V = \left(\frac{\partial}{\partial T}\left[\underbrace{\frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]}_{h(T)}\right]\right)_V$$

$$f(T) \to \frac{1}{T} \qquad g(T) \to \left(\frac{\partial U}{\partial V}\right)_T + P$$

$$= \frac{\partial f}{\partial T}\cdot g(T) + \frac{\partial g}{\partial T}\cdot f(T)$$

$$= \underline{-\frac{1}{T^2}\left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] + \frac{1}{T}\left[\left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T\right)_V\right]}$$

$$\parallel$$

$$\to \cancel{\frac{1}{T}\left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right)_T} = -\frac{1}{T^2}\left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] + \frac{1}{T}\left[\left(\frac{\partial P}{\partial T}\right)_V + \cancel{\left(\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T\right)_V}\right]$$

$$0 = -\frac{1}{T^{\cancel{2}}}\left[\underline{\left(\frac{\partial U}{\partial V}\right)_T} + P\right] + \cancel{\frac{1}{T}}\left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \boxed{\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P} \; *$$