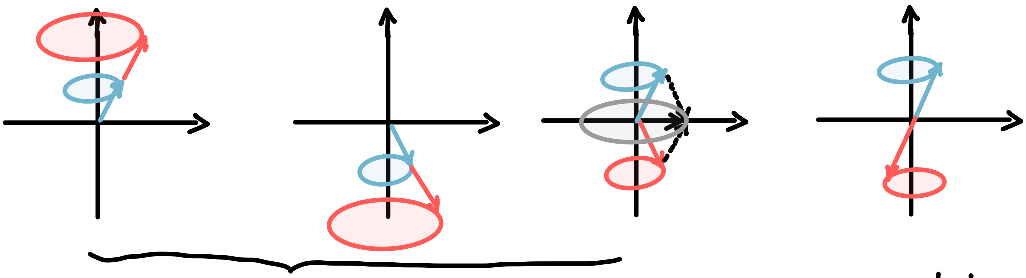
Triplet & Singlet
$(\uparrow\uparrow)$ $(\uparrow\downarrow)$



1.6 Correlation Amplitude.

Definition: $C(t) \equiv \langle \underset{|\alpha;t_0\rangle}{\underline{\alpha}} | \alpha, t_0; t\rangle$
$= \langle \alpha | \hat{u}(t,0) | \alpha \rangle$

How much $|\alpha, t_0; t\rangle$ resemble the initial state $|\alpha\rangle$

$\underline{|C(t)|}$ quantifies the similarity.

1> if $|\alpha\rangle = |a'\rangle$

$\Rightarrow C(t) = \langle a' | \underline{a', t_0; t\rangle} = \langle a' | \underline{e^{-\frac{i}{\hbar}E_{a'}t} | a'\rangle}$

$= e^{-\frac{i}{\hbar}E_{a'}t}$

$\Rightarrow |C(t)| = 1 \Rightarrow$ stationary state.

$\Rightarrow$ if $|\alpha\rangle = \sum_{a'} C_{a'} |a'\rangle$

$$C_{a'} \downarrow \langle a'|\alpha\rangle$$

$$\Rightarrow C(t) = \langle \alpha ; t_0 | \alpha , t_0 ; t\rangle$$

$$= \left(\sum_{a'} C_{a'}^* \langle a'|\right)\left(\sum_{a''} \underbrace{e^{-\frac{i}{\hbar}E_{a''}t} C_{a''}}_{C_{a''}(t)} |a''\rangle\right)$$

$$= \sum_{a'} C_{a'}^* C_{a'} e^{-\frac{i}{\hbar}E_{a'}t} \underbrace{\langle a'|a'\rangle}_{\Downarrow \; 1}$$

$$= \sum_{a'} |C_{a'}|^2 e^{-\frac{i}{\hbar}E_{a'}t}$$

Now if $[\hat{A}, \hat{H}] = 0$ $\quad \hat{A}|a'\rangle = a'|a'\rangle$

$\hat{H}$ has a continum spectra for energy eigenvalues.

$$\underline{\hat{H}|a'\rangle = E_{a'}|a'\rangle}$$

$$\Downarrow$$

re-write: $\hat{H}|\tilde{E}\rangle = E|\tilde{E}\rangle$

$$C_{a'} = \langle a'|\alpha\rangle = \langle \tilde{E}|\alpha\rangle = C(E)$$

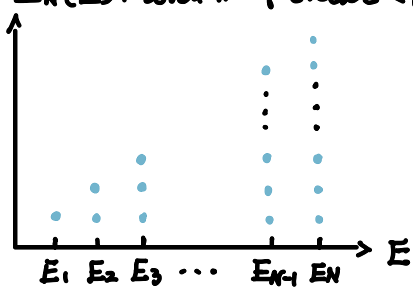
$$\Rightarrow C(t) = \int \underset{\downarrow a'}{dE} \, |\underset{\downarrow a'}{C(E)}|^2 \, e^{-\frac{i}{\hbar}\underset{\downarrow E_{a'}}{E}t}$$

$|E_1\rangle \quad |E_2\rangle \cdots |E_N\rangle$

$E_1 \quad E_2 \cdots E_N \cdots$

$$\Rightarrow \rho(E) \sim \frac{dZ_N(E)}{dE}$$

$Z_N(E)$: total # of states $\leq E_N$



1> Energy eigen values are evenly distributed
No degeneracy.

$\Rightarrow \rho(E) = \tilde{1}$ (const)

2> if NOT case 1> $\Rightarrow \rho(E) \neq$ const.

$\Rightarrow C(t) = \int dE\, |C(E)|^2 \rho(E)\, e^{-\frac{i}{\hbar}Et}$

Degeneracy : $g(E) = \lim\limits_{\Delta E \to 0} \int_E^{E+\Delta E} \rho(E)\, dE$

Further Discussion related to spectral line shapes.

$f(E) = |C(E)|^2 \rho(E)$

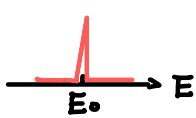
How the quatum state is spread across different energy eigenstate.

$$\Rightarrow \quad C(t) = \int dE\, f(E)\, e^{-\frac{i}{\hbar}Et}.$$

*. Narrow vs. Broad Energy distribution.

1> Narrow : if distribution is sharply peaked at E_0

$\Rightarrow$



$\Rightarrow \Delta E \to 0$

$\Rightarrow f(E) \simeq \delta(E - E_0) \Rightarrow C(t) = \int dE\, \delta(E-E_0)\, e^{-\frac{i}{\hbar}Et}$

Oscillation
& coherence (long-lived)

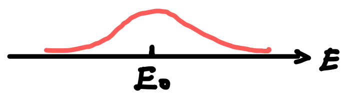
$= 1 \cdot e^{-\frac{i}{\hbar}E_0 t}$

e.g., A single atomic sys w/o interaction exhibits perfect coherence, leading to Rabi oscillation.

2> Broad Energy Distribution.

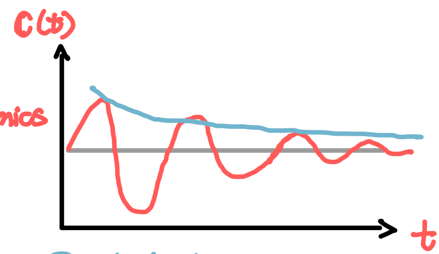
case 1: Gaussian distribution ⇒ Dephasing.

$$f(E) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(E-E_0)^2}{2\sigma^2}}$$



$$\Rightarrow C(t) = e^{-\frac{i}{\hbar}E_0 t}\, e^{-\frac{\sigma^2 t^2}{2\hbar^2}}$$

⇒ Dynamics

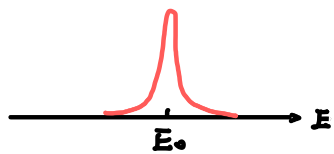


case 2: Lorentzian Energy Distribution

$$f(E) = \frac{\Gamma/\pi}{(E-E_0)^2+\Gamma^2}$$

Decay rate

$$\Rightarrow C(t) = e^{-\frac{i}{\hbar}E_0 t}\, e^{-\frac{\Gamma}{\hbar}t}$$



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2. The Schrödinger vs. Heisenberg picture.

1> Schrödinger: $\hat{U}(t, t_0)$ affect $|\alpha; t_0\rangle$

$|\alpha, t_0; t\rangle \Rightarrow i\hbar \frac{\partial}{\partial t}|\alpha, t_0; t\rangle = \hat{H}|\alpha, t_0; t\rangle$

2> Heisenberg picture: observables vary w/ time.

2-1. Unitary operator

$$\hat{T}(dx'),\quad \hat{u}(t,t_0)\ \Big\}\longrightarrow \hat{U}$$

if $|\alpha'\rangle = \hat{U}|\alpha\rangle \qquad |\beta'\rangle = \hat{U}|\beta\rangle$

$$\Rightarrow \langle\beta'|\alpha'\rangle = \langle\hat{U}\beta|(\hat{U}\alpha)\rangle = \langle\beta|\underbrace{\hat{U}^{+}\hat{U}}_{\Downarrow\ \mathbb{1}}|\alpha\rangle = \langle\beta|\alpha\rangle$$

Now $\langle\beta'|\hat{x}|\alpha'\rangle$

$= \langle\hat{U}\beta|\hat{x}|(\hat{U}\alpha)\rangle$

1.) $= (\langle\beta|\hat{U}^{+})\,\hat{x}\,(\hat{U}|\alpha\rangle)$ $\longrightarrow$ Schrödinger

2.) $= \langle\beta|(\hat{U}^{+}\hat{x}\hat{U})|\alpha\rangle$ $\longrightarrow$ Heisenberg.

$\hat{U}^{+}\hat{x}\hat{U}$ $\longrightarrow$ New observable (changes)