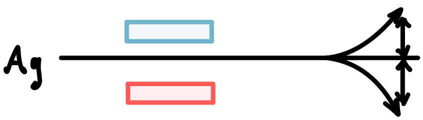
$$\Rightarrow \quad \hat{S}_x = ?\,(|x;+\rangle\langle x;+|) + ?'(|x;-\rangle\langle x;-|)$$

$$= ?\left(\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}(1 \quad 1)\right) + ?'\left(\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}(1 \quad -1)\right)$$

$$= \frac{?}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} + \frac{?'}{2}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix} \qquad ? = \frac{\hbar}{2}, \quad ?' = -\frac{\hbar}{2}$$



$$F_z = \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B})$$

$$\downarrow$$

$$\mu \propto S \Rightarrow \frac{\hbar}{2}$$

$$\Rightarrow \hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$

$$\Rightarrow \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \qquad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned}\hat{\sigma}_1 &= \hat{\sigma}_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\\ \hat{\sigma}_2 &= \hat{\sigma}_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}\\ \hat{\sigma}_3 &= \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\end{aligned}}$$

$\Rightarrow$ Pauli Matrices.

$$\mathbb{1} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$\{\mathbb{1}, \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ forms a 4D space that spans all 2×2 Hermitian matrices.

Obj. #2. Quantum Dynamics.

1. Time evolution & schrödinger Equation.
2. schrödinger VS. Heisenberg Picture.
3. Harmonic Oscillator (review)
4. Time-dependent Wave Equations.
5. Feynman Path Integrals
6. Gauge Transformation.

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1. Time evolution & schrödinger Equation.

1. Time-evolution operator.

$|\alpha\rangle \longrightarrow$ Initially at t_0

$\longrightarrow |\alpha; t_0\rangle \; (\sim |\alpha\rangle)$

later at $t \longrightarrow |\alpha, t_0 ; t\rangle$

$$\longrightarrow \lim_{t\to 0} |\alpha, t_0 ; t\rangle = |\alpha, t_0 ; t_0\rangle = |\alpha ; t_0\rangle \; (\sim |\alpha\rangle)$$

Analogy: $\hat{T}(\Delta x)|x'\rangle = |x' + \Delta x\rangle$

$$\underbrace{\hat{u}(t, t_0)}_{\text{Time Evolution Operater.}} |\alpha ; t_0\rangle = |\alpha, t_0 ; t\rangle$$

* Properties of $\hat{u}(t, t_0)$

1> $$|\alpha; t_0\rangle = \sum_{a'} \underbrace{|a'\rangle}_{\{|a'\rangle\} \to \text{Basis set.}} \underbrace{\langle a'|\alpha, t_0\rangle}_{c_{a'}(t_0)}$$

$$\Rightarrow |\alpha, t_0 ; t\rangle = \sum_{a'} c_{a'}(t)\, |a'\rangle$$

typically: $c_{a'}(t) \neq c_{a'}(t_0)$

However: $$\sum_{a'} |c_{a'}(t_0)|^2 = \sum_{a'} |c_{a'}(t)|^2$$

i.e. if $\langle \alpha ; t_0 | \alpha ; t_0\rangle = 1 \Rightarrow \langle \alpha, t_0 ; t | \alpha, t_0 ; t\rangle = 1$

$$\langle \hat{u}(t, t_0)\alpha ; t_0 \,|\, \hat{u}(t, t_0)\alpha ; t_0\rangle$$

$$\langle \alpha ; t_0 | \hat{u}^{+} \; \hat{u} | \alpha ; t_0\rangle$$

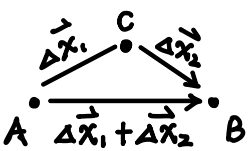
$=$

$$\Rightarrow \boxed{\hat{u}^{+}\hat{u} = \mathbb{1}} \; *$$

$\Downarrow$

unitary.

2> Recall



$$\hat{T}(\Delta x_2)\,\hat{T}(\Delta x_1) = \hat{T}(\Delta x_1 + \Delta x_2)$$

likewise, $t_0 \longrightarrow t_1 \longrightarrow t_2$

$$\boxed{\begin{array}{c} \hat{u}(t_2, t_0) = \hat{u}(t_2, t_1)\,\hat{u}(t_1, t_0) \\ t_2 \geq t_1 \geq t_0 \end{array}} \; *$$

3> $\lim_{\Delta x \to 0} \hat{T}(\Delta x) = \mathbb{1}$ likewise, $\lim_{\Delta t \to 0} \hat{U}(t_0+\Delta t, t_0) = \mathbb{1}$

4>

$$\begin{tikzcd}
|\alpha; t_0\rangle \arrow[r, "\hat{U}(t,t_0)"'] \arrow[rr, bend left, "\mathbb{1}"] & |\alpha, t_0; t\rangle \arrow[r, "\hat{U}(t_0,t)"'] & |\alpha, t_0; t_0\rangle
\end{tikzcd}$$

$\Rightarrow \underbrace{\hat{U}(t,t_0)}_{}^{\hat{A}} \underbrace{\hat{U}(t_0,t)}_{}^{\hat{B}} = \mathbb{1} = \hat{U}^{+}(t,t_0)\hat{U}(t,t_0)$

$\Rightarrow \hat{U}^{+}(t,t_0) = \hat{U}(t_0,t)$

likewise $\Rightarrow \underbrace{\hat{U}(t_0,t)}_{\hat{B}} \underbrace{\hat{U}(t,t_0)}_{\hat{A}} = \mathbb{1}$.

$\Rightarrow \boxed{\hat{U}(t_0,t) = \hat{U}^{-1}(t,t_0)}$ *

$\Rightarrow \boxed{\hat{U}(t_0,t) = \hat{U}^{+}(t,t_0) = \hat{U}^{-1}(t,t_0)}$ *.

$\rightarrow$ reversing time.

$\Rightarrow$ recall $\hat{T}(dx') = \mathbb{1} - i\hat{K}dx'$

$\downarrow$

$\hat{K}^{+} = \hat{K}$ (Proof Based $\hat{T}^{+}\hat{T} = \mathbb{1}$.

$\hat{T}(-dx')\hat{T}(dx') = \mathbb{1}$)

Analogously $\Rightarrow \hat{U}(t_0+dt, t_0) = \mathbb{1} - i\hat{\Omega}dt$

$\rightarrow \hat{\Omega}^{+} = \hat{\Omega} \Rightarrow$ Herm.

for example: $\Rightarrow \hat{u}^{+}\hat{u} = \mathbb{1}$.

$$\hat{u}^{+}(t_0+dt, t_0)\,\hat{u}(t_0+dt, t_0) = (\mathbb{1} + i\hat{\Omega}^{+}dt)(\mathbb{1} - i\hat{\Omega}dt) \cong \mathbb{1}$$

$\hat{K}$ in $\hat{T}$ is related $\hat{P}$

$\hat{T} = \mathbb{1} - i\hat{K}dx' \Rightarrow \hat{K}$ in [cm^{-1}]

$\hat{P}$ in [$\underbrace{J\cdot S}_{\hbar\ [J\cdot S]\ \text{or}\ [J\cdot Hz^{-1}]}\cdot cm^{-1}$]

$\Rightarrow \hat{K} = \dfrac{\hat{P}}{\hbar}$

$$\Rightarrow \hat{T} = \mathbb{1} - \frac{i}{\hbar}\hat{P}dx'$$

likewise. $\hat{u}(t_0+dt, t_0) = \mathbb{1} - i\underbrace{\hat{\Omega}}_{[S^{-1}]}\overbrace{dt}^{[S]}$

$\boxed{E = \hbar\omega = h\nu}$ Plank – Einstein Relation.

$$\frac{h}{2\pi}\,\frac{2\pi}{T} = \frac{h}{T} = \frac{h}{\lambda/c} = h\nu$$

$\underbrace{\tilde{\hat{\Omega}}}_{\text{Frequency}} = \dfrac{\tilde{\hat{E}}}{\hbar}$ $\qquad \hat{\Omega} = \dfrac{\hat{H}}{\hbar} \Rightarrow \hat{u}(t_0+dt, t_0) = \mathbb{1} - \dfrac{i}{\hbar}\overbrace{\hat{H}}^{\text{Herm.}}dt$